Available online at www.sciencedirect.com
ScienceDirect
Journal of Applied Mathematics and Mechanics 71 (2007) 905–913
Journal of APPLIED MATHEMATICS AND MECHANICS
www.elsevier.com/locate/jappmathmech

Self-excited oscillations of a two-mass oscillator with dry "stick-slip" friction☆

S.V. Khizgiyayev

Moscow, Russia

Received 15 March 2007

Abstract

A system of two masses, moving along a single straight line, is considered. The first is connected by a spring to a fixed point, while the second is connected by a spring to the first and is in contact with a belt with dry friction moving with constant velocity. A piecewise-constant model of dry friction with different coefficients of friction, sliding and at rest, is used. The limit "stick-slip" type cycles are investigated analytically. It is shown numerically that in the case of equal masses there are forward and reverse limit cycles. The period of the oscillations of the forward and reverse cycles increases as the ratio of the stick and slip coefficients of friction increases, and decreases when the velocity of the belt increases. The reverse cycle exists for all values of the parameters of the problem, while the forward cycle exists up to a certain critical value of the ratio of the stick and slip coefficients of friction, and this critical value increases when the velocity of the belt increases.

A similar problem was investigated in Ref. 1 in the case when the amplitude of the oscillations is constant, there is a rational ratio between the frequencies and for a piecewise-cubic model of the friction, and a method was proposed for deriving approximate analytic averaged equations. A piecewise-cubic model of the friction was also considered in Refs. 2–5.

1. Formulation of the problem

Consider a system of two loads with masses m_1 and m_2. The load m_1 is connected to a fixed wall by a weightless linear spring with stiffness k_1, while the load m_2 is connected to m_1 by a spring of stiffness k_2. The load m_2 is placed on a horizontal belt, which moves with constant velocity $\tilde{v} \geq 0$. The displacements of the loads from the position in which the springs are undeformed will be denoted by $\tilde{x}_1$ and $\tilde{x}_2$ (Fig. 1). When $\tilde{x}_1 = \tilde{x}_2 = 0$, when there is no friction force, the system is in a stable position of equilibrium. We will use the piecewise-constant model of dry friction with a coefficient of friction f between the belt and the load m_2,[6–10] in

☆ Prikl. Mat. Mekh. Vol. 71, No. 6, pp. 1004–1013, 2007.
E-mail address: simen@simen.ru.

0021-8928/$ – see front matter
doi:10.1016/j.jappmathmech.2007.12.009

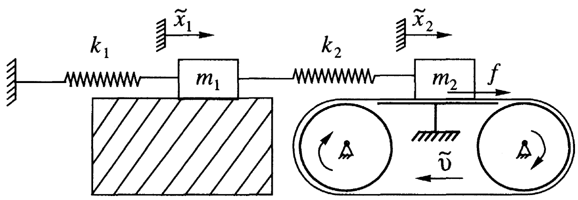


Fig. 1.

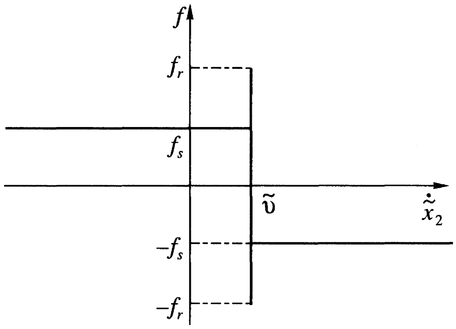


Fig. 2.

the form (Fig. 2)

$$f = \begin{cases} f_s \text{ For } \dot{\tilde{x}}_2 < \tilde{\upsilon} \\ f_s \text{ For } \dot{\tilde{x}}_2 = \tilde{\upsilon} \text{ and } f_* > f_r \\ f_* \text{ For } \dot{\tilde{x}}_2 = \tilde{\upsilon} \text{ and } |f_*| \le f_r; \quad f_* = k_2(\tilde{x}_2 - \tilde{x}_1) \\ -f_s \text{ For } \dot{\tilde{x}}_2 = \tilde{\upsilon} \text{ and } f_* < -f_r \\ -f_s \text{ For } \dot{\tilde{x}}_2 > \tilde{\upsilon} \end{cases} \tag{1.1}$$

A dot denotes a derivative with respect to $\tilde{t}$, f_s and $f_r (f_r \geq f_s)$ are the slip and stick coefficients of friction, proportional to the normal reaction. The force of dry friction is a function of the difference of the velocities $\dot{x}_2 - v$ and the difference of the coordinates $x_2 - x_1$.

We will change to dimensionless quantities, choosing as the characteristic units for measuring time, mass and length $\sqrt{m_1/(k_1 + k_2)}$, m$_2$ and $f_s/(k_1 + k_2)$. The dimensionless variables are expressed in terms of the old variables as follows:

$$t = \sqrt{\frac{k_1 + k_2}{m_1}}\tilde{t}, \quad x_i = \frac{k_1 + k_2}{f_s}\tilde{x}_i, \quad \upsilon = \frac{\sqrt{m_1(k_1 + k_2)}}{f_s}\tilde{\upsilon}$$

In dimensionless variables, the equations of motion of the system in matrix form are

$$\ddot{X} + KX = F$$
$$X = \left\| \begin{matrix} x_1 \\ x_2 \end{matrix} \right\|, \quad F = \left\| \begin{matrix} 0 \\ \eta f/f_s \end{matrix} \right\|, \quad K = \left\| \begin{matrix} 1 & -\chi \\ -\chi\eta & \chi\eta \end{matrix} \right\|, \quad \chi = \frac{k_2}{k_1 + k_2}, \quad \eta = \frac{m_1}{m_2} \tag{1.2}$$

Depending on which of the conditions $\dot{x}_2 < v$, $\dot{x}_2 > v$ or $\dot{x}_2 = v$ is satisfied, the motion of the system corresponds to one of three different modes, each of which is described by a system of linear inhomogeneous differential equations.

2. Linear modes of motion

We will first construct a general solution of the homogeneous Eq. (1.2) (with $F = 0$). Using the linear transformation

$$\begin{Vmatrix} x_1 \\ x_2 \end{Vmatrix} = \begin{Vmatrix} \sqrt{\chi}e_+ & -\sqrt{\chi}e_- \\ \sqrt{\chi\eta}e_- & \sqrt{\chi\eta}e_+ \end{Vmatrix} \begin{Vmatrix} z_1 \\ z_2 \end{Vmatrix}; \quad e_\pm = \sqrt{\frac{\sqrt{d} \pm \chi\eta \mp 1}{2\sqrt{d}}}$$

$$d = (\chi\eta - 1)^2 + 4\chi^2\eta$$

we change to normal coordinates $Z = (z_1, z_2)^T$, in which the homogeneous Eq. (1.2) takes the form

$$\ddot{Z} + \Omega Z = 0; \quad \Omega = \begin{Vmatrix} \omega_-^2 & 0 \\ 0 & \omega_+^2 \end{Vmatrix}, \quad \omega_\pm^2 = \frac{1 + \chi\eta \pm \sqrt{d}}{2} \tag{2.1}$$

Hence it follows that $\omega_- \neq \omega_+$ for any values of the parameters, since otherwise this leads to the contradictory conditions $\chi\eta = 1$ and $\chi^2\eta = 0$.

In the original coordinates X, the solution of Eq. (2.1) has the form

$$\begin{Vmatrix} X(t, P) \\ \dot{X}(t, P) \end{Vmatrix} = \begin{Vmatrix} \Gamma_1(t) & \Gamma_2(t) \\ \Gamma_3(t) & \Gamma_1(t) \end{Vmatrix} P, \quad P = \begin{Vmatrix} X_0 \\ \dot{X}_0 \end{Vmatrix}$$

$$\Gamma_i(t) = \Lambda G_i(t)\Lambda^{-1}, \quad i = 1, 2, 3, \quad G_1(t) = \begin{Vmatrix} c_- & 0 \\ 0 & c_+ \end{Vmatrix}$$

$$G_2(t) = \begin{Vmatrix} s_-/\omega_- & 0 \\ 0 & s_+/\omega_+ \end{Vmatrix}, \quad G_3(t) = \begin{Vmatrix} -s_-\omega_- & 0 \\ 0 & -s_+\omega_+ \end{Vmatrix} \tag{2.2}$$

$$\Lambda = \begin{Vmatrix} e_+ & -e_- \\ \sqrt{\eta}e_- & \sqrt{\eta}e_+ \end{Vmatrix}, \quad \Lambda^{-1} = \begin{Vmatrix} e_+ & e_-/\sqrt{\eta} \\ -e_- & e_+/\sqrt{\eta} \end{Vmatrix}$$

$$c_\pm = \cos(\omega_\pm t), \quad s_\pm = \sin(\omega_\pm t)$$

We will now consider the solution of the inhomogeneous systems of equations for the modes $\dot{x}_2 < 2$ and $\dot{x}_2 > 2$. The solutions $X^+(t)$ and $X^-(t)$ for the modes $\dot{x}_2 < v$ and $\dot{x}_2 > v$ are given by the equations

$$\ddot{X}^\pm + KX^\pm = (0, \pm\eta)^T \tag{2.3}$$

with the choice of the plus and minus signs respectively. The particular stationary solution of these systems has the form

$$X_s^\pm = \pm h, \quad h = (h_1, h_2)^T, \quad h_1 = \frac{1}{1-\chi}, \quad h_2 = \frac{1}{\chi(1-\chi)} \tag{2.4}$$

Solutions of systems (2.3) are obtained in the form of the sum of the particular solution and the general solution (2.2) of the homogeneous system (1.2) (for $F = 0$)

$$\begin{Vmatrix} X^\pm(t, Q) \\ \dot{X}^\pm(t, Q) \end{Vmatrix} = \begin{Vmatrix} \Gamma_1(t) & \Gamma_2(t) \\ \Gamma_3(t) & \Gamma_1(t) \end{Vmatrix} \begin{Vmatrix} X_0^\pm - X_s^\pm \\ \dot{X}_0^\pm \end{Vmatrix} + \begin{Vmatrix} X_s^\pm \\ 0 \end{Vmatrix}; \quad Q = \begin{Vmatrix} X_0^\pm \\ \dot{X}_0^\pm \end{Vmatrix} \tag{2.5}$$

When $\dot{x}_2 = v$ when the condition for remaining in this mode is satisfied

$$|x_2(t) - x_1(t)| \le \mu/\chi, \quad \mu = f_r/f_s \tag{2.6}$$

the system loses one degree of freedom and its motion is given by the equations

$$\ddot{x}_1 + x_1 - \chi x_2 = 0, \quad \dot{x}_2 = v \tag{2.7}$$

the general solution of which in matrix form is

$$\begin{Vmatrix} X^0(t,R) \\ \dot{X}^0(t,R) \end{Vmatrix} = \begin{Vmatrix} H_1(t) & H_2(t) \\ H_3(t) & H_1(t) \end{Vmatrix} R, \quad R = \begin{Vmatrix} X_0^0 \\ \dot{X}_0^0 \end{Vmatrix}, \quad X^0(t,R) = \begin{Vmatrix} x_1^0(t) \\ x_2^0(t) \end{Vmatrix}$$

$$H_i(t) = \Sigma D_i(t) \Sigma^{-1}, \quad i = 1, 2, 3$$

$$D_1(t) = \begin{Vmatrix} \cos t & 0 \\ 0 & 1 \end{Vmatrix}, \quad D_2(t) = \begin{Vmatrix} \sin t & 0 \\ 0 & t \end{Vmatrix} \tag{2.8}$$

$$D_3(t) = \begin{Vmatrix} -\sin t & 0 \\ 0 & 0 \end{Vmatrix}, \quad \Sigma = \begin{Vmatrix} 1 & \chi \\ 0 & 1 \end{Vmatrix}, \quad \Sigma^{-1} = \begin{Vmatrix} 1 & -\chi \\ 0 & 1 \end{Vmatrix}$$

We will trace the change in the linear modes during the motion of the system. Suppose that at the initial instant $t=0$ the condition $\dot{x}_2(0) > v$ is satisfied, and at the instant $t=t_1$ the condition $\dot{x}_2(t_1) > v$ holds.

If $x_2(t_1) - x_1(t_1) > \mu/\chi$, the velocity of the second body with respect to the motion of the belt changes sign ($\dot{x}_2 < v$) and motion begins by virtue of the system of differential equations (2.3) in the opposite direction to the motion of the belt. This motion will continue until the instant $t=t_2$, $\dot{x}_2(t_2) = v$. Then, when the condition $x_2(t_2) - x_1(t_2) \ge -\mu/\chi$ is satisfied, the further motion will be described by system (2.7) and when $x_2(t_2) - x_1(t_2) < -\mu/\chi$ it will be described by system (2.3).

If at the instant of time $t=t_1$ the condition $x_2(t_1) - x_1(t_1) \le \mu/\chi$ is satisfied, the second body will move together with the belt, by virtue of system (2.7). This motion will continue up to the instant $t=t_3$, $x_2(t_3) - x_1(t_3) = \mu/\chi$. The system then returns to the mode $\dot{x}_2 < v$, and so on.

3. The fundamental property of the limit cycle

We will consider the conditions for the simplest limit cycle, consisting of two successive modes of linear motion, to exist. Suppose the phase trajectory of this cycle (Fig. 3) has a period $T + \tau$ and successively passes through the points

$$A = (X(0), \dot{X}(0))^T, \quad B = (X(T), \dot{X}(T))^T, \quad C = (X(T+\tau), \dot{X}(T+\tau))^T$$

where the following limitations are imposed on the position and velocity of the limit cycle

$$x_2(0) - x_1(0) = \mu/\chi, \quad \dot{x}_2(0) = \dot{x}_2(T) = v \tag{3.1}$$

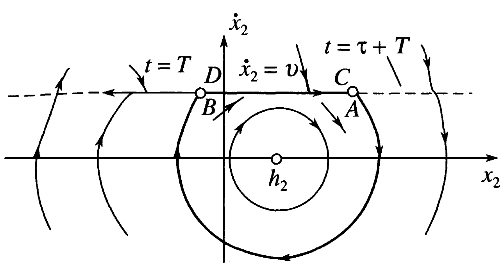


Fig. 3.

Then the arc AB is determined by the solution $X^+(t)$ when $0<t\leq T$, and the arc BC is determined by the solution $X^0(t-T)$ when $T\leq t\leq T+\tau$.

Taking the autonomy of system (2.7) into account, the point

$$B = D = (x^0(-\tau), \dot{x}^0(-\tau))^T$$

can also be obtained by reverse motion along the arc CB along the solution $X_0(t)$ (2.8) with initial condition $(X(0), \dot{X}(0))^T$. Assuming that both motions begin at the point A, the condition $B=D$ for the limit cycle to be periodic can be written in the form

$$(X^+(T), \dot{X}^+(T))^T = (X^0(-\tau), \dot{X}^0(-\tau))^T$$

In expanded form, by virtue of relations (2.5) and (2.8), the last condition has the form

$$\begin{Vmatrix} \Gamma_1 & \Gamma_2 \\ \Gamma_3 & \Gamma_1 \end{Vmatrix} \begin{Vmatrix} X(0)-h \\ \dot{X}(0) \end{Vmatrix} + \begin{Vmatrix} h \\ 0 \end{Vmatrix} = \begin{Vmatrix} H_1 & -H_2 \\ -H_3 & H_1 \end{Vmatrix} \begin{Vmatrix} X(0) \\ \dot{X}(0) \end{Vmatrix} \tag{3.2}$$

where $H_i=H_i(\tau)$, $\Gamma_i=\Gamma_i(T)$ $(i=1,2,3)$.

We will prove the following fundamental property of the limit cycle

$$X_B-h = -(X_A-h), \quad \dot{X}_B = \dot{X}_A \tag{3.3}$$

or

$$X^+(T)-h = -(X(0)-h), \quad \dot{X}^+(T) = \dot{X}(0) \tag{3.4}$$

We will prove property (3.3) in homogeneous variables

$$Y(t) = X(t)-h, \quad Y_A = (y_1(0), y_2(0))^T, \quad \dot{Y}_A = (\dot{y}_1(0), \upsilon)^T \tag{3.5}$$

in which condition (3.3) takes the form

$$Y_B = -Y_A, \quad \dot{Y}_B = \dot{Y}_A \tag{3.6}$$

The condition of periodicity of the limit cycle $B=D$ (3.2) in the variables (3.5) takes the form

$$\begin{Vmatrix} \Gamma_1 Y_A+\Gamma_2\dot{Y}_A \\ \Gamma_3 Y_A+\Gamma_1\dot{Y}_A \end{Vmatrix} = \begin{Vmatrix} H_1 Y_A-H_2\dot{Y}_A+(H_1-E)h \\ -H_3(Y_A+h)+H_1\dot{Y}_A \end{Vmatrix} \tag{3.7}$$

By virtue of relations (2.4) and (2.8) we have

$$(H_1-E)h = 0, \quad H_3 h = 0 \tag{3.8}$$

Then, condition (3.7) can be rewritten in the form of a system of equations in $(Y_A, \dot{Y}_A)$

$$\begin{aligned} (\Gamma_1-H_1)Y_A+(\Gamma_2+H_2)\dot{Y}_A &= 0 \\ (\Gamma_3+H_3)Y_A+(\Gamma_1-H_1)\dot{Y}_A &= 0 \end{aligned} \tag{3.9}$$

Similarly, starting the construction of the limit cycle from the point B in the phase space $(Y, \dot{Y})$, we can write equations for the initial conditions at the point $B=(Y_B, \dot{Y}_B)^T$. To do this we transfer the origin of the reading of time to the point B. Then, when $0\leq t\leq\tau$, by virtue of the autonomy of system (2.7), the limit cycle has the form

$$\begin{Vmatrix} Y(t) \\ \dot{Y}(t) \end{Vmatrix} = \begin{Vmatrix} H_1(t) & H_2(t) \\ H_3(t) & H_1(t) \end{Vmatrix} \begin{Vmatrix} Y_B+h \\ \dot{Y}_B \end{Vmatrix} - \begin{Vmatrix} h \\ 0 \end{Vmatrix}$$

When $\tau \le t \le \tau + T$ the limit cycle has the form

$$\begin{Vmatrix} Y(t,B) \\ \dot{Y}(t,B) \end{Vmatrix} = \begin{Vmatrix} \Gamma_1(t) & \Gamma_2(t) \\ \Gamma_3(t) & \Gamma_1(t) \end{Vmatrix} B$$

We will write the condition for the periodicity of the limit cycle $A = C$ in the form

$$\begin{Vmatrix} \Gamma_1 Y_B - \Gamma_2 \dot{Y}_B \\ -\Gamma_3 Y_B + \Gamma_1 Y_B \end{Vmatrix} = \begin{Vmatrix} H_1(Y_B + h) + H_2 \dot{Y}_B - h \\ H_3(Y_B + h) + H_1 \dot{Y}_B \end{Vmatrix} \tag{3.10}$$

By virtue of relations (3.8) we can rewrite (3.10) in the form of a system of equations in $(Y_B, \dot{Y}_B)$

$$\begin{aligned} -(\Gamma_1 - H_1)Y_B + (\Gamma_2 + H_2)\dot{Y}_B &= 0 \\ -(\Gamma_3 + H_3)Y_B + (\Gamma_1 - H_1)\dot{Y}_B &= 0 \end{aligned} \tag{3.11}$$

Note that when $(Y_B, \dot{Y}_B)$ is replaced by $(-Y_A, \dot{Y}_A)$, the system of equations (3.11) becomes (3.9). Suppose $(Y, \dot{Y})$ is the solution of system (3.9). Then, in view of the last property, $(-lY, l\dot{Y})$ is the solution of system (3.11) where l is a certain non-zero constant. By virtue of the fact that the second components $\dot{Y}$ and $l\dot{Y}$ of the solutions of systems (3.9) and (3.11) are the same and $\dot{y}_{2A} = \dot{y}_{2B} = v$ in view of condition (3.1), we have $l = 1$. Then, $Y_B = -Y_A$, $\dot{Y}_B = \dot{Y}_A$ holds for the solutions of systems (3.9) and (3.11), which also proves relation (3.3).

4. Periodic solutions and limit cycles

If, for specified χ, η the condition $\dot{x}_2 < v$ is satisfied for any t, the system can only be in one mode (2.3). In this case there are two principal oscillations of the system about the equilibrium position $(x_1, x_2) = (h_1, h_2)$ with frequencies $\omega_-/(2\pi)$ and $\omega_+/(2\pi)$ and initial conditions defined by the conditions

$$\dot{x}_2(0)e_\pm = \pm\sqrt{\eta}\dot{x}_1(0)e_\mp, \quad (x_2(0) - h_2)e_\pm = \mp\sqrt{\eta}(x_1(0) - h_1)e_\mp \tag{4.1}$$

In the limiting case, when the trajectory of the principal oscillations in phase space is in contact with the plane $\dot{x}_2 < v$, the initial conditions are defined by the relations

$$x_1(0) = h_1, \quad x_2(0) = h_2, \quad \dot{x}_1(0) = \pm\frac{ve_\pm}{\sqrt{\eta}e_\mp}, \quad \dot{x}_2(0) = v \tag{4.2}$$

The principal oscillations in the limiting case (4.2) only exist when $h_2 - h_1 = \mu/\chi$, i.e. when $\mu = 1$.

We will obtain the limit cycles with two reversals. For these we express $x_1(0)$, $x_2(0)$, $\dot{x}_1(0)$ in terms of τ and T using properties (3.1) and (3.3), which we rewrite in the form

$$\begin{aligned} &-(x_1(0) - h_1) = x_1(-\tau) - h_1, \quad -(x_2(0) - h_2) = x_2(-\tau) - h_2 \\ &\dot{x}_1(0) = \dot{x}_1(-\tau), \quad x_2(0) = x_1(0) + \mu/\chi \end{aligned} \tag{4.3}$$

Eq. (4.3) are not independent. If we express $x_1(0)$, $x_2(0)$, $\dot{x}_1(0)$ from the last three equations

$$\begin{aligned} &x_1(0) = \frac{v\tau}{2} + \frac{1}{\chi(1-\chi)} - \frac{\mu}{\chi}, \quad x_2(0) = \frac{v\tau}{2} + \frac{1}{\chi(1-\chi)} \\ &\dot{x}_1(0) = \chi v + \left[\frac{v\tau}{2}(1-\chi) + \frac{1-\mu}{\chi}\right]\operatorname{ctg}\frac{\tau}{2} \end{aligned} \tag{4.4}$$

and substitute into the first equation of (4.3), we obtain an identity.

Property (3.4), by virtue of relation (2.2) and the last expression, is equivalent to the following system of two equations:

$$(\Gamma_{11}+1)\left(\frac{\tau}{2}-\sigma\right)+\Gamma_{12}\frac{\tau}{2}+\Gamma_{21}\chi_0+\Gamma_{22} = 0$$
$$\Gamma_{13}\left(\frac{\tau}{2}-\sigma\right)+(\Gamma_{14}+1)\frac{\tau}{2}+\Gamma_{23}\chi_0+\Gamma_{24} = 0 \quad (4.5)$$
$$\sigma = \frac{\mu-1}{\chi\upsilon},\quad \chi_0 = \chi+\left[\frac{\tau(1-\chi)}{2}-\sigma\right]\mathrm{ctg}\frac{\tau}{2},\quad \Gamma_i = \{\Gamma_{ij}(T)\}$$

where $\Gamma_{ij}=\Gamma_{ij}(T)$ ($j=1, 2, 3, 4, i=1, 2, 3$) are the elements of the matrix Γ_i. Eq. (4.5) depend only on τ, T, σ, χ and form a closed system of equations in τ and T. Here τ can be expressed from Eq. (4.5) as a function of T

$$\tau = \frac{2(\Gamma_{22}\Gamma_{23}-\Gamma_{24}\Gamma_{21})-2(\Gamma_{23}(\Gamma_{11}+1)-\Gamma_{13}\Gamma_{21})\sigma}{(\Gamma_{13}+\Gamma_{14}+1)\Gamma_{21}-(\Gamma_{11}+\Gamma_{12}+1)\Gamma_{23}} \quad (4.6)$$

Substituting expression (4.6) into the first equation of (4.5), we obtain, for fixed χ and ε, the equation $\Phi(T, \sigma)=0$.

We will obtain the limit cycles for $\chi=0.6$ and $\varepsilon=1$ numerically. From the solutions of the equation $\Phi(T, \sigma)=0$ we select only two limit cycles: the forward (slow) and reverse (fast) cycles. The first limit cycle is the forward cycle in the sense that the masses m_1 and m_2 at the reversal point A have accelerations of equal sign, i.e. $\ddot{x}_1(0) < 0$, $\ddot{x}_2(0) < 0$ (Fig. 4), and is slow in the sense that when $\sigma=0$ it is produced from the principal oscillation with initial conditions (4.2) when the upper sign is chosen, with the lowest frequency $\omega_-/(2\pi)$. The second limit cycle is a reverse cycle in the sense that the masses m_1 and m_2 at the reversal point A have accelerations of opposite sign, i.e. $\ddot{x}_1(0) > 0$, $\ddot{x}_2 < 0$ (Fig. 5), and is fast in the sense that when $\sigma=0$, it is produced from the principal oscillation with initial conditions (4.2), when the lower sign is chosen with frequency $\omega_+/(2\pi)$ to be greater than $\omega_-/(2\pi)$ by virtue of the definition of $\omega_\pm$ (2.1).

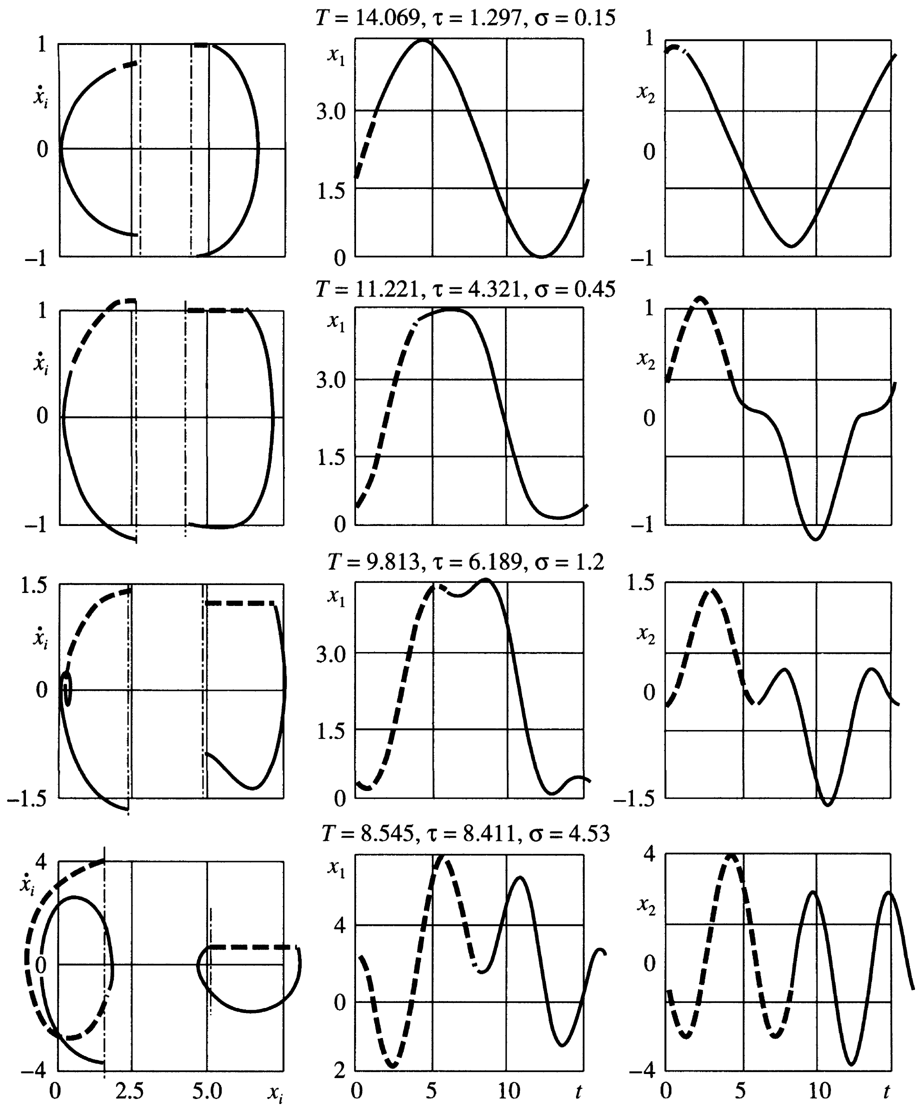
It has been shown numerically that the phase trajectory of the limit cycles obtained in four-dimensional phase space is symmetrical about the plane formed by the straight lines $x_1=h_1$ and $x_2=h_2$, i.e. the projection of the phase trajectory onto the plane $(x_1, \dot{x}_1)$ and $(x_2, \dot{x}_2)$ is symmetrical about the straight lines $x_1=h_1\approx 2.5$ and $x_2=h_2\approx 4.17$, represented by the dash-dot curve. In Figs. 4 and 5 we show half of the projections of the phase trajectories onto the $(x_1, \dot{x}_1)$ and $(x_2, \dot{x}_2)$ planes. The left curve corresponds to the projections of the phase trajectory onto the $(x_1, \dot{x}_1)$ plane, while the right curve corresponds to the projection onto the $(x_2, \dot{x}_2)$ plane, where the dashes indicate the part of the trajectory corresponding to the mode $x_2 = v$.

In Fig. 6 we show the form of the curve $\Phi(T, \sigma)=0$ for the forward cycle (the right-hand curve) and the reverse cycle (the left-hand curve). Other solutions of the equation $\Phi(T, \sigma)=0$ are not considered.

It has been shown numerically that for the reverse cycle, when σ increases, the value of T decreases (Fig. 6), while the value of τ and the period $\tau+T$ increase. The reverse cycle exists for all values of σ and its form does not change when σ changes. When $\sigma\to\infty$, the value of T approaches 0, and the value of τ and the period of the limit cycle approach 2π.

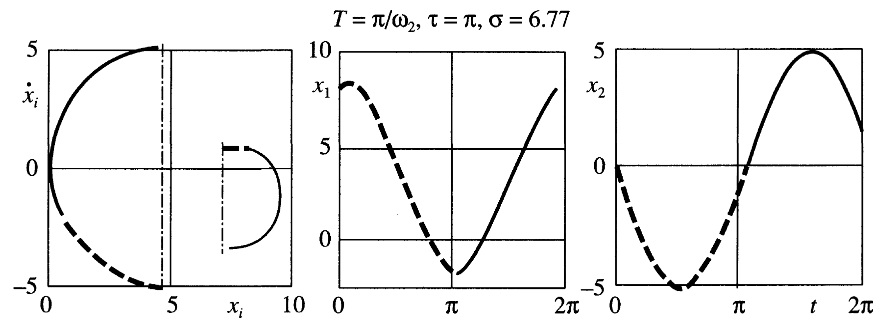
For the forward cycle it has been shown numerically that when σ increases the value of T decreases (Fig. 6), while τ and the period $\tau+T$ increase. For small σ the form of the oscillations of the forward cycle is close to sinusoidal in shape (Fig. 4, $\sigma\approx 0.15$). When σ increases there will be an oscillation with twice the frequency in the forward cycle, as a result of which in the (t,x_2) plane there will be four additional points of inflection (Fig. 4, $\sigma\approx 0.45$), and then, as σ increases, the oscillation with double frequency becomes predominant (Fig. 4, $\sigma\approx 1.2$, 4.53).

The forward cycle exists up to a critical value $\sigma_*\approx 4.53$ and disappears when $\sigma>\sigma_*$. This occurs by virtue of the fact that in the $(x_2, \dot{x}_2)$ plane when σ is close to σ_*, a loop appears which rises to the section $\dot{x}_2 = v$ as σ increases. When $\sigma=\sigma_*$ this loop reaches $\dot{x}_2 = v$, and the limit cycle degenerates into oscillations which are not a limit cycle in view of the breakdown of conditions (3.3). Since $\sigma=(\mu-1)/\chi v$, the results obtained for $\chi=0.6$ and $\eta=1$ indicate that the reverse cycle exists for all values of x, μ, v. For large ratios between the slip and stick coefficients of friction the forward cycle disappears, and the critical value of this difference increases as the velocity of the belt v increases.



T = 14.069, τ = 1.297, σ = 0.15
T = 11.221, τ = 4.321, σ = 0.45
T = 9.813, τ = 6.189, σ = 1.2
T = 8.545, τ = 8.411, σ = 4.53

Fig. 4.



T = π/ω2, τ = π, σ = 6.77

Fig. 5.

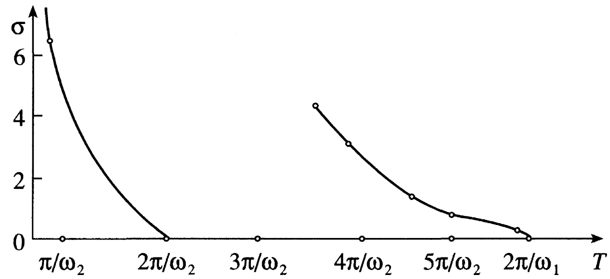


Fig. 6.

Acknowledgements

I wish to thank S. Ya. Stepanov for his help in formulating and solving the problem and M. Pascal for her comments.

This research was supported financially by the Russian Foundation for Basic Research (05-01-00454) and the Programme for the Support of the Leading Scientific Schools (NSh-6667.2006.1).

References

1. Awrejcewicz J. Analysis of sele-excited vibration due to dry friction in a system with two degrees of freedom. *Sci Bull Lodz Tech Univ* 1993;**82**(635):921–31.
2. Awrejcewicz J, Dzyubak L. Regular and chaotic Motion of a non-linear quasiautonomous oscillator with Coulomb and viscous friction. *Visn Tekh Univ Podilya* 2003;**Pt 1, 1**(56):56–66.
3. Liu C-S, Chang W-T. Frictional behaviour of a belt-driven and periodically excited oscillator. *J Sound and Vibrat* 2002;**258**(2):247–68.
4. Awrejcewicz J, Dzyubak L, Grebogi C. Estimation of chaotic and regular (stick-slip and slip-slip) oscillations exhibited by coupled oscillators with dry friction. *Nonlinear Dynamics* 2005;**42**(4):383–94.
5. Awrejcewicz J, Olejnik P. Friction pair modeling by a 2-dof system; numerical and experimental investigations. *Int J Bifurcat and Chaos Special Issue* 2005;**15**(6):1931–44.
6. Andronov AA, Vitt AA, Khaikin SE. *Theory of Oscillations*. Oxford: Pergamon; 1966.
7. Andreaus U, Casini P. Dynamics of fraction oscillators excited by a moving base and/or driving force. *J Sound and Vibrat* 2001;**245**(4):685–99.
8. Khizgiyayev SV. Analytic investigation of if a self-excited oscillatory mechanical system with a piecewise-constant model of dry friction. In: *Problems of Investigating the Stability and Stabilization of Motion*. Moscow: VTS Ross Akad Nauk; 2006. p. 84–92.
9. Pascal M. dynamics and stability of a two degrees of freedom oscillator with an elastic stop. *J Comput and Nonlinear Dynamics* 2006;**1**(1):94–102.
10. Pascal M., Stepanov S.Ya. Periodic solutions and stability of piecewise linear vibratory systems. *Selected Proceedings of the 4th Polyakov Lectures*, St. Petersburg, 2006. C. 84–94.

Translated by R.C.G.